Tutorial 1

The 1PN expression for the GW energy flux in terms of the mass quadrupole (I_{ij}), mass octupole (I_{ijk}) and current quadrupole moment (J_{ij}) is written within the MPM-PN approach as

$$\mathcal{F} = \frac{G}{c^5}\left\{\frac{1}{5}(I_{ij}^{(3)})^2 + \frac{1}{c^2}\left[\frac{1}{189}(I_{ijk}^{(4)})^2 + \frac{16}{45}(J_{ij}^{(3)})^2\right] + \mathcal{O}\left(\frac{1}{c^4}\right)\right\}$$

Here the PN counting refers to powers of $1/c$ – 1PN refers to corrections of the order $1/c^2$, 2PN of the order $1/c^4$ etc.

P1. Starting with 1PN expression for mass quadrupole (Eq. 4.4a) and leading order expressions for mass octupole and current quadrupole moment (Eq. 4.4b and Eq 4.4f) compute expressions for their respective derivatives listed in **Appendix C** of Reference [2] – ideally a MATHEMATICA code utilizing tensor package although the computations can also be done by hand albeit may become lengthy for higher order derivatives. Note that the notation $x_{\langle ij\rangle} \equiv x_{\langle i}x_{j\rangle}$ or $\mathrm{STF}_{ij}(x_i x_j)$ indicates that the the quantity $x_i x_j$ is **symmetric and trace free** in the two indices.

P2. Use the above to construct squares – $(I_{ij}^{(3)})^2, (I_{ijk}^{(4})^2, (J_{ij}^{(3)})^2$ at 1PN provided as Eqs. (C4a), (C4b) and (C4d) of **Appendix C** of Reference [2].

P3. Use these in the equation above to obtain 1PN expression for the flux given as

$$\mathcal{F} = \frac{32\nu^2}{5}\frac{G^4 m^5}{c^5 r^5}\left[1 - \frac{Gm}{c^2 r}\left(\frac{2927}{336} + \frac{5}{4}\nu\right) + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

Here $m = m_1 + m_2$ (total mass) and $\nu = \mu m$ where μ a the reduced mass.

References

1. https://arxiv.org/abs/1310.1528
2. https://arxiv.org/abs/gr-qc/9501029
3. https://arxiv.org/abs/gr-qc/0010009